While Gauss refers to two earlier synthetic constructions of Paucker,[1] as the only previous ones which he knew had been publicly discussed, he remarks that that of Erchinger is "different and carried through more in the spirit of pure geometry."

Stäckel reviews the discussions of the division of a circle into n equal parts, by means of imaginary quantities and the solution of the equation $x^n - 1 = 0$, by Cotes,[2] De Moivre,[3] Euler,[4] and Vandermonde,[5] and the development of their ideas by Gauss in connection with the problem of constructing a regular inscribed polygon.

PROBLEMS FOR SOLUTION.

[N.B. The editorial work of this department would be greatly facilitated if, on sending in problems, the proposers would also enclose their solutions—*when they have them.* If a problem proposed is not original the proposer is requested *invariably* to state the fact and to give an exact reference to the source.]

2843. Proposed by E. H. MOORE, University of Chicago.

Show that the maximum of the absolute value of

$$2(a + ib)(x + iy) + i(a + ib)(z + iw) + i(c + id)(x + iy),$$

where $i = \sqrt{-1}$ and a, b, c, d, x, y, z, w are real numbers for which

$$a^2 + b^2 + c^2 + d^2 = x^2 + y^2 + z^2 + w^2 = 1,$$

is $1 + \sqrt{2}$. Study the locus of point-pairs $P = (a, b, c, d)$, $Q = (x, y, z, w)$ of the unit-sphere in real four-space for which this absolute value assumes its maximum value.

2844. Proposed by J. L. RILEY, Stephenville, Texas.

Decompose into simple fractions the number $\frac{6099380351}{1271808720}$ (Gauss, *Disq. Arith.*, *Werke*, vol. 1, pp. 386–387).

2845. Proposed by E. L. POST, Princeton University.

Prove that if y_x is a solution of the functional equation

$$y_x = \frac{y_{x+1}^2}{x} + y_{x+1}$$

for positive integral values of x with $y_x > 0$, then

$$\lim_{x \to \infty} y_x \log x = 1.$$

2846.

Find the entire volume within the surface $x^{1/2} + y^{1/2} + z^{1/2} = a^{1/2}$. (W. A. Granville, *Elements of Differential and Integral Calculus*, revised ed., 1911, p. 420.)

This equation, rationalized, is the equation of Steiner's quartic surface, every tangent plane to which cuts it in two conics. (Cf. Salmon-Rogers, *Analytic Geometry of Three Dimensions*, 5th ed., vol. 2, 1915, pp. 171, 201, 207, 213f. Also C. M. Jessop, *Quartic Surfaces* 1916, Chapter 7.)

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[1] (1) "Geometrische Verzeichnung des regelmässigen 17-Ecks und 257-Ecks in d. Kreis," *Jahresverhandl. d. kurländischen Gesellschaft für Litteratur und Kunst*, Mitau, Band 2, 1822. (2) *Die ebene Geometrie der geraden Linie und des Kreises*. Königsberg, 1823, p. 187. Paucker is also the author of: (3) *De divisione geometrica peripheriae circuli in XVII partes æquales*, Königsberg, 1817.

[2] R. Cotes, *Harmonia mensurarum, sive analysis et synthesis per rationum et angulorum mensuras promota*. Cambridge, 1722.

[3] A. De Moivre, *Miscellanea analytica*, London, 1730.

[4] L. Euler, *Introductio in analysin*, Lausanne, 1748, especially t. 1, cap 8: De quantitatibus transcendentibus ex circulo ortis.

[5] C. A. Vandermonde, "Remarques sur les problèmes de situation," *Histoire de l'Acad.*, année 1771, Paris 1774; *Mémoires*, p. 566. "Sur la résolution des équations," p. 365.